Práctica 0 (1)

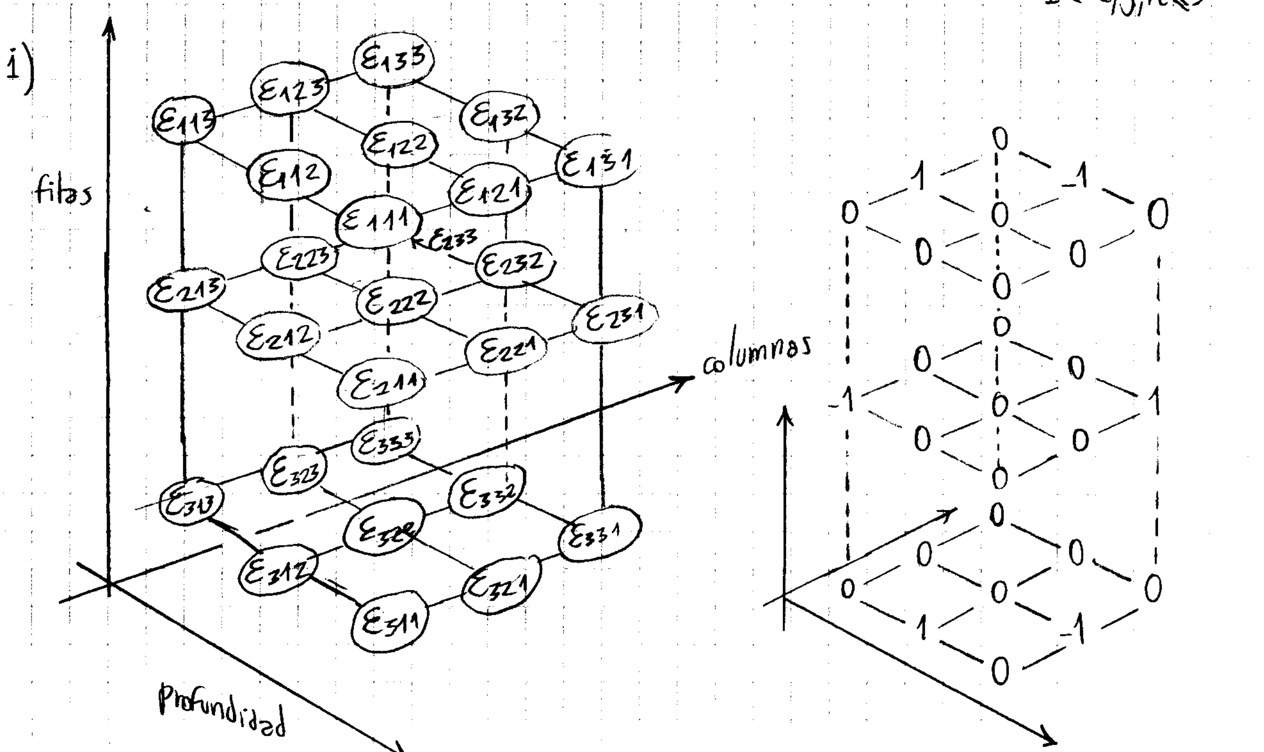
1.

$$\delta_{ij} = \begin{cases} 1 & \text{si } i=j \\ 0 & \text{si } i\neq j \end{cases} \qquad 1\leq i,j\leq 3$$

$$\varepsilon_{ijk} = \begin{cases} 1 & \text{si } \{i,j,k\} \text{ son permutación cíclica de } \{1,2,3\} \\ -1 & \text{si } \{i,j,k\} \text{ son permutación anticíclica de } \{1,2,3\} \\ 0 & \text{si dos índices son iguales} \end{cases}$$

$1\leq i,j,k\leq 3$

i)



La densidad tensorial ε_{ijk} tiene 27 elementos

iii)

e) $\delta_{mn}\delta_{mn} = \delta_{mm}\delta_{mm} = 1.1 + 1.1 + 1.1 = 3$

d)
$$\varepsilon_{ijk}\varepsilon_{ijk} = \sum_k \sum_j \varepsilon_{1jk}\varepsilon_{1jk} + \varepsilon_{2jk}\varepsilon_{2jk} + \varepsilon_{3jk}\varepsilon_{3jk}$$
$$= \sum_k \varepsilon_{12k}\varepsilon_{12k} + \varepsilon_{13k}\varepsilon_{13k} + \varepsilon_{21k}\varepsilon_{21k} + \varepsilon_{23k}\varepsilon_{23k} + \varepsilon_{31k}\varepsilon_{31k} + \varepsilon_{32k}\varepsilon_{32k}$$
$$= \varepsilon_{123}.\varepsilon_{123} + \varepsilon_{132}.\varepsilon_{132} + \varepsilon_{213}.\varepsilon_{213} + \varepsilon_{231}.\varepsilon_{231} + \varepsilon_{312}\varepsilon_{312} + \varepsilon_{321}\varepsilon_{321}$$
$$= 1.1 + (-1).(-1) + (-1)(-1) + 1.1 + 1.1 + (-1).(-1)$$
$$\varepsilon_{ijk}\varepsilon_{ijk} = 6$$

c)
$$\varepsilon_{ijk}\varepsilon_{ijl} = \sum_i^3 \sum_j^3 \varepsilon_{ijk}\varepsilon_{ijl}$$
$$= \varepsilon_{12k}\varepsilon_{12l} + \varepsilon_{13k}\varepsilon_{13l} + \varepsilon_{21k}\varepsilon_{21l} + \varepsilon_{23k}\varepsilon_{23l} + \varepsilon_{31k}\varepsilon_{31l} + \varepsilon_{32k}\varepsilon_{32l}$$

$= \varepsilon_{12k}(\varepsilon_{12l} - \varepsilon_{21l}) + \varepsilon_{31k}(\varepsilon_{31l} - \varepsilon_{13l}) + \varepsilon_{23k}(\varepsilon_{23l} - \varepsilon_{32l})$

$= \varepsilon_{12k}\, 2\, \varepsilon_{12l} + \varepsilon_{31k}\, 2\, \varepsilon_{31l} + \varepsilon_{23k}\, 2\, \varepsilon_{23l}$

$= 2(\underbrace{\varepsilon_{12k}\cdot\varepsilon_{12l}}_{3} + \underbrace{\varepsilon_{31k}\cdot\varepsilon_{31l}}_{2} + \underbrace{\varepsilon_{23k}\,\varepsilon_{23l}}_{1}) = 2 \sum_{k,l} \delta_{kl} \Rightarrow$

213
123

$\boxed{\varepsilon_{ijk}\,\varepsilon_{ijl} = 2\delta_{kl}}$ (con convención de Einstein metida)

∂) $\varepsilon_{ijk}\,\varepsilon_{irs} = \sum_i \varepsilon_{ijk}\,\varepsilon_{irs} = \varepsilon_{1jk}\,\varepsilon_{1rs} + \varepsilon_{2jk}\,\varepsilon_{2rs} + \varepsilon_{3jk}\,\varepsilon_{3rs}$

$= \varepsilon_{1jk}\,\varepsilon_{1rs} - \varepsilon_{j2k}\,\varepsilon_{2rs}$

ii)

$$\varepsilon_{ijk}\cdot\varepsilon_{pqr} = \begin{vmatrix} \delta_{ip} & \delta_{iq} & \delta_{ir} \\ \delta_{jp} & \delta_{jq} & \delta_{jr} \\ \delta_{kp} & \delta_{kq} & \delta_{kr} \end{vmatrix}$$

$$= \delta_{ip}\delta_{jq}\delta_{kr} - \delta_{ir}\delta_{jq}\delta_{kp} + \delta_{jp}\delta_{kq}\delta_{ir} - \delta_{jr}\delta_{kq}\delta_{ip} + \delta_{kp}\delta_{iq}\delta_{jr} - \delta_{kr}\delta_{iq}\delta_{jp}$$

$$= \delta_{jq}(\delta_{ip}\delta_{kr} - \delta_{ir}\delta_{kp}) + \delta_{kq}(\delta_{jp}\delta_{ir} - \delta_{jr}\delta_{ip}) + \delta_{iq}(\delta_{kp}\delta_{jr} - \delta_{kr}\delta_{jp})$$

$$= \delta_{jq}[\varepsilon_{lik}\,\varepsilon_{lpr}] + \delta_{kq}[\varepsilon_{lji}\cdot\varepsilon_{lpr}] + \delta_{iq}[\varepsilon_{l}$$

3. $\vec{U}, \vec{V}, \vec{W}, \vec{S}$ vectores ψ, ϕ campos escalares

i) $\vec{U}\cdot(\vec{V}\times\vec{W}) = \vec{V}\cdot(\vec{W}\times\vec{U}) = \vec{W}\cdot(\vec{U}\times\vec{V})$ ← (es un escalar)

permutando cíclicamente

$$U_i\, \varepsilon_{ijk}\, V_j\, W_k = V_j\, \varepsilon_{jki}\, W_k\, U_i = W_k\, \varepsilon_{kij}\, U_i\, V_j$$

$$\boxed{\vec{U}\cdot(\vec{V}\times\vec{W}) = \vec{V}\cdot(\vec{W}\times\vec{U}) = \vec{W}\cdot(\vec{U}\times\vec{V})}$$

ii) $\left(\vec{U}\times \underbrace{[\vec{V}\times\vec{W}]}_{=\vec{B}}\right)_i \rightarrow$ es un vector $\quad b_k = \varepsilon_{klm} V_l W_m$

$$\varepsilon_{ijk}\, U_j\, b_k = \varepsilon_{ijk}\, U_j\, \varepsilon_{klm} V_l W_m$$

$$= \delta_{il}\delta_{jm} U_j V_l W_m - \delta_{im}\delta_{jl} U_j V_l W_m$$

$$\delta_{il} V_l\, \delta_{jm} U_j W_m - \delta_{im} W_m\, \delta_{jl} U_j V_l$$

$$\boxed{\left[\vec{U}\times(\vec{V}\times\vec{W})\right] = \vec{V}(\vec{U}\cdot\vec{W}) - \vec{W}(\vec{U}\cdot\vec{V})}$$

iii) $(\vec{U}\times\vec{V})\cdot(\vec{W}\times\vec{S}) \rightarrow$ es un escalar

$$\varepsilon_{ijk}\, U_j V_k\, \varepsilon_{ipq} W_p S_q = \delta_{jp}\delta_{kq} U_j V_k W_p S_q - \delta_{jq}\delta_{kp} U_j V_k W_p S_q$$

$$= \delta_{jp} U_j W_p\, \delta_{kq} V_k S_q - \delta_{jq} U_j S_q\, \delta_{kp} V_k W_p$$

$$\boxed{(\vec{U}\times\vec{V})(\vec{W}\times\vec{S}) = (\vec{U}\cdot\vec{W})(\vec{V}\cdot\vec{S}) - (\vec{U}\cdot\vec{S})(\vec{V}\cdot\vec{W})}$$

iv) $\vec{\nabla}\cdot\vec{r} = 3 \qquad \vec{r} = (x, y, z) \qquad$ La divergencia es un escalar

$$\frac{\partial r_i}{\partial x_i} = \frac{\partial r_x}{\partial x} + \frac{\partial r_y}{\partial y} + \frac{\partial r_z}{\partial z} = 1+1+1 = 3 \rightarrow \boxed{\frac{\partial r_i}{\partial x_i} = 3}$$

v) $\vec{\nabla}\times\vec{r}$

$$(\vec{\nabla}\times\vec{r})_i = \varepsilon_{ijk}\frac{\partial r_k}{\partial x_j} = 0 \rightarrow \boxed{\vec{\nabla}\times\vec{r} = \vec{0} = 0}$$

vi) $\vec{\nabla} r = \frac{\vec{r}}{r}$ ↘ un vector $\qquad \sqrt{x_i^2 + x_j^2 + x_k^2}$

$$(\vec{\nabla} r)_i = \frac{\partial r}{\partial x_i} = \frac{x_i}{r} \Rightarrow \vec{\nabla} r = \left(\frac{x_1}{r}, \frac{x_2}{r}, \frac{x_3}{r}\right) \Rightarrow \boxed{\vec{\nabla} r = \frac{\vec{r}}{r}}$$

vii) $\vec{\nabla}\left(\frac{1}{r}\right) \qquad \frac{1}{r} = (x_1^2 + x_2^2 + x_3^2)^{-1/2} \qquad -\frac{1}{\cancel{2}}\frac{\cancel{2}x_i}{r^3}$

$$\left[\vec{\nabla}\left(\frac{1}{r}\right)\right]_i = \frac{\partial(1/r)}{\partial x_i} = \frac{-x_i}{r^3} \Rightarrow \boxed{\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}}$$

viii) $\vec{\nabla}\times\vec{\nabla}\phi = 0 \longrightarrow$ vector

$$[\vec{\nabla}\times\vec{\nabla}\phi]_i = \varepsilon_{ijk}\frac{\partial(\vec{\nabla}\phi)_k}{\partial x_j} = \varepsilon_{ijk}\frac{\partial}{\partial x_j}\cdot\frac{\partial\phi}{\partial x_k} = \varepsilon_{ijk}\frac{\partial^2\phi}{\partial x_j\partial x_k}$$

$$= \frac{\partial^2\phi}{\partial x_j\partial x_k} - \frac{\partial^2\phi}{\partial x_k\partial x_j} = 0 \Rightarrow$$

$$\boxed{\vec{\nabla}\times(\vec{\nabla}\phi) = 0}$$

ix) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{U}) = 0 \rightsquigarrow$ escalar

$$\frac{\partial}{\partial x_i}\varepsilon_{ijk}\frac{\partial U_k}{\partial x_j} = \varepsilon_{ijk}\frac{\partial^2 U_k}{\partial x_i\partial j} = \varepsilon_{jki}\frac{\partial^2 U_i}{\partial x_j\partial x_k} = \varepsilon_{kij}\frac{\partial^2 U_j}{\partial x_k\partial x_i}$$

$$= \varepsilon_{123}\frac{\partial^2 U_3}{\partial x_1\partial x_2} + \varepsilon_{231}\frac{\partial^2 U_1}{\partial x_2\partial x_3} + \varepsilon_{312}\frac{\partial^2 U_2}{\partial x_3\partial x_1}$$

$$-\varepsilon_{132}\frac{\partial^2 U_2}{\partial x_1\partial x_2} - \varepsilon_{213}\frac{\partial^2 U_3}{\partial x_2\partial x_1} - \varepsilon_{321}\frac{\partial^2 U_1}{\partial x_3\partial x_2}$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{U}) = 0}$$

x) $\nabla^2\Psi = \vec{\nabla}\cdot(\vec{\nabla}\Psi) \rightsquigarrow$ es un escalar

$$\frac{\partial^2\Psi}{\partial x_j^2} = \frac{\partial}{\partial x_j}\left(\frac{\partial\Psi}{\partial x_j}\right) = \vec{\nabla}\cdot(\vec{\nabla}\Psi)$$

xi) $\nabla^2(\phi\Psi) \rightsquigarrow$ es un escalar

$$\frac{\partial^2(\phi\Psi)}{\partial x_j^2} = \frac{\partial}{\partial x_j}\left(\frac{\partial\phi}{\partial x_j}\cdot\Psi + \phi\cdot\frac{\partial\Psi}{\partial x_j}\right)$$

$$= \frac{\partial^2\phi}{\partial x_j^2}\cdot\Psi + \frac{\partial\phi}{\partial x_j}\cdot\frac{\partial\Psi}{\partial x_j} + \frac{\partial\phi}{\partial x_j}\cdot\frac{\partial\Psi}{\partial x_j} + \phi\cdot\frac{\partial^2\Psi}{\partial x_j^2}$$

$$= \nabla^2\phi\cdot\Psi + \vec{\nabla}\phi\cdot\vec{\nabla}\Psi + \vec{\nabla}\phi\cdot\vec{\nabla}\Psi + \phi\cdot\nabla^2\Psi$$

$$\nabla^2(\phi\Psi) = \nabla^2\phi\cdot\Psi + 2\,\vec{\nabla}\phi\,\vec{\nabla}\Psi + \phi\cdot\nabla^2\Psi$$

$$\boxed{\Delta^2(\phi\Psi) = \Delta^2\phi\cdot\Psi + 2\,\mathrm{grad}(\phi)\,\mathrm{grad}(\Psi) + \phi\cdot\Delta^2\Psi}$$

xii) $\vec{\nabla}(\phi\Psi) = \frac{\partial}{\partial x_i}[\phi\Psi]\,\hat{e}_i$

es un vector

$$\{\vec{\nabla}(\phi\Psi)\}_i = \frac{\partial\phi}{\partial x_i}\Psi + \frac{\partial\Psi}{\partial x_i}\cdot\phi \Rightarrow$$

$$\vec{\nabla}(\phi\Psi) = \vec{\nabla}\phi\cdot\Psi + \vec{\nabla}\Psi\cdot\phi \Rightarrow \boxed{\mathrm{grad}(\phi\Psi) = \Psi\cdot\mathrm{grad}\,\phi + \phi\cdot\mathrm{grad}\,\Psi}$$

xiii) $\vec{\nabla}\cdot(\vec{U}\times\vec{V}) =$ $\longrightarrow$ es un escalar

$$\frac{\partial}{\partial x_i}\{\vec{U}\times\vec{V}\}_i = \frac{\partial}{\partial x_i}\varepsilon_{ijk}U_jV_k$$

$$= \varepsilon_{ijk}\frac{\partial U_j}{\partial x_i}.V_k + \varepsilon_{ijk}U_j.\frac{\partial V_k}{\partial x_i}$$

$$= (\vec{\nabla}\times\vec{U}).\vec{V} - \vec{U}.(\vec{\nabla}\times\vec{V})$$

$$\boxed{\operatorname{div}(\vec{U}\times\vec{V}) = \vec{V}.\operatorname{rot}(\vec{U}) - \vec{U}.\operatorname{rot}(\vec{V})}$$

xiv) $\vec{\nabla}.(\phi\vec{U}) =$ $\longrightarrow$ un escalar

$$\frac{\partial(\phi U_i)}{\partial x_i} = \frac{\partial\phi}{\partial x_i}.U_i + \phi.\frac{\partial U_i}{\partial x_i}$$

$$\vec{\nabla}.(\phi\vec{U}) = (\vec{\nabla}\phi).\vec{U} + \phi(\vec{\nabla}.\vec{U})$$

$$\boxed{\operatorname{div}(\phi\vec{U}) = \vec{U}.\operatorname{grad}(\phi) + \phi.\operatorname{div}(\vec{U})}$$

xv) $\vec{\nabla}\times(\phi\vec{U}) = \varepsilon_{ijk}\frac{\partial(U_k\phi)}{\partial x_j}$

es un vector

$$\{\vec{\nabla}\times(\phi\vec{U})\}_i = \varepsilon_{ijk}\frac{\partial U_k}{\partial x_j}.\phi + \varepsilon_{ijk}\frac{\partial\phi}{\partial x_j}.U_k$$

$$\vec{\nabla}\times(\phi\vec{U}) = \phi(\vec{\nabla}\times\vec{U}) + (\vec{\nabla}\phi\times\vec{U})$$

$$\boxed{\operatorname{rot}(\phi\vec{U}) = \phi.\operatorname{rot}(\vec{U}) + \operatorname{grad}(\phi)\times\vec{U}}$$

xvi) $\vec{\nabla}\times(\vec{U}\times\vec{V}) =$ $\longrightarrow$ es un vector

$$(\vec{\nabla}\times\vec{U}\times\vec{V})_i = \varepsilon_{ijk}\frac{\partial[\vec{U}\times\vec{V}]_k}{\partial x_j} = \varepsilon_{ijk}\frac{\partial}{\partial x_j}(\varepsilon_{kpq}U_pV_q)$$

$\varepsilon_{ijk}\varepsilon_{irs} = \delta_{jr}\delta_{ks} - \delta_{js}\delta_{kr}$

$\varepsilon_{jki}\varepsilon_{irs} =$

$\varepsilon_{jki}\varepsilon_{rsi} =$

$$(\quad)_k = \varepsilon_{kij}\frac{\partial}{\partial x_i}(\varepsilon_{jpq}U_pV_q)$$

$$(\quad)_k = \varepsilon_{kij}\varepsilon_{jpq}\left(\frac{\partial U_p}{\partial x_i}V_q + U_p.\frac{\partial V_q}{\partial x_i}\right)$$

$$= \varepsilon_{kij}\varepsilon_{pqj}\left(\frac{\partial}{\partial x_i}U_p V_q + U_p.\frac{\partial V_q}{\partial x_i}\right)$$

si permuto k pierdo el carácter vectorial del ente $\Rightarrow$ conviene permuto el segundo Levi-Civita

$$= (\delta_{kp}\delta_{iq} - \delta_{kq}\delta_{ip})\left(\frac{\partial U_p}{\partial x_i}.V_q + U_p.\frac{\partial V_q}{\partial x_i}\right)$$

$$(\quad)_k = \frac{\partial U_k}{\partial x_i}.V_i - \frac{\partial U_i}{\partial x_i}V_k + U_k\frac{\partial V_i}{\partial x_i} - U_i\frac{\partial V_k}{\partial x_i}$$

$$= V_i\frac{\partial}{\partial x_i}U_k - \frac{\partial U_i}{\partial x_i}.V_k + U_k.\frac{\partial V_i}{\partial x_i} - U_i\frac{\partial V_k}{\partial x_i}$$

$$\boxed{\vec{\nabla}\times(\vec{U}\times\vec{V}) = (\vec{V}.\vec{\nabla})\vec{U} - (\vec{\nabla}.\vec{U})\vec{V} + \vec{U}(\vec{\nabla}.\vec{V}) - (\vec{U}.\vec{\nabla})\vec{V}}$$

$$\varepsilon_{ijm}\,\varepsilon_{ike} = \delta_{jk}\delta_{me} - \delta_{je}\delta_{mk}$$
$$= \delta_{jk}\delta_{ke} - \delta_{je} \quad m=k$$

xvii) $\vec{\nabla}(\vec{U}.\vec{V}) =$ matriz; es un vector

$$\{\vec\nabla(\vec U.\vec V)\}_i = \frac{\partial}{\partial x_i} U_j V_j = \frac{\partial U_i}{\partial x_i} V_j + U_j \frac{\partial V_j}{\partial x_i} =$$ → no se puede expresar fácil

$$= \frac{\partial}{\partial x_i}\delta_{jk} U_j V_k = \delta_{je}\frac{\partial U_j}{\partial x_i} V_e + \delta_{je} U_j \frac{\partial V_e}{\partial x_i}$$

$$= (\delta_{jk}\delta_{ke} - \varepsilon_{ijk}\varepsilon_{ike})\left(\frac{\partial U_j}{\partial x_i} V_e + U_j\frac{\partial V_e}{\partial x_i}\right)$$

$$= \delta_{jk}\delta_{ke} V_e \frac{\partial U_j}{\partial x_i} + \delta_{jk}\delta_{ke} U_j \frac{\partial V_e}{\partial x_i} - \varepsilon_{ijk}\varepsilon_{ike}\frac{\partial U_j}{\partial x_i} V_e - \varepsilon_{ijk}\varepsilon_{ike} U_j \frac{\partial V_e}{\partial x_i}$$

$$V_j \frac{\partial U_j}{\partial x_i} + U_j\frac{\partial V_j}{\partial x_i} - \varepsilon_{ike}\left(\varepsilon_{kij}\frac{\partial U_j}{\partial x_i}\right) V_e + \varepsilon_{ijk} U_j\, \varepsilon_{kie}\frac{\partial V_e}{\partial x_i}$$

$$= (\vec V.\vec\nabla)\vec U + (\vec U.\vec\nabla)\vec V - (\vec\nabla\times\vec U)\times\vec V + \vec U\times(\vec\nabla\times\vec V)$$

$$\boxed{\mathrm{grad}(\vec U.\vec V) = (\vec V.\mathrm{grad})\vec U + (\vec U.\mathrm{grad})\vec V + \vec V\times\mathrm{rot}(\vec U) + \vec U\times\mathrm{rot}(\vec V)}$$

xviii) $\nabla^2\vec U = \vec\nabla\cdot(\vec\nabla\vec U) = \frac{\partial}{\partial x_i}(\vec\nabla\vec U)_i = \frac{\partial}{\partial x_i}\left(\frac{\partial U_i}{\partial x_j}\right) = \frac{\partial}{\partial x_i}\left(\delta_{ik}\frac{\partial U_k}{\partial x_j}\right)$

Vector; vector; matriz

$$= \delta_{ik}\frac{\partial}{\partial x_i}\left(\frac{\partial U_k}{\partial x_j}\right) =$$

$$\vec\nabla U_1 = \left(\frac{\partial U_1}{\partial x_1}, \frac{\partial U_1}{\partial x_2}, \frac{\partial U_1}{\partial x_3}\right)$$

$$= (\delta_{il}\delta_{lk} - \varepsilon_{jil}\varepsilon_{jlk})\frac{\partial}{\partial x_i}\left(\frac{\partial U_k}{\partial x_j}\right)$$

$$\vec\nabla\vec U = \begin{pmatrix}\partial_1 U_1 & \partial_1 U_2 & \partial_1 U_3\\ \partial_2 U_1 & \partial_2 U_2 & \partial_2 U_3\\ \partial_3 U_1 & \partial_3 U_2 & \partial_3 U_3\end{pmatrix} = (\partial_i U_j) = (\vec\nabla\vec U)_j$$

$$= \delta_{il}\delta_{lk}\frac{\partial}{\partial x_i}\left(\frac{\partial U_k}{\partial x_j}\right) - \varepsilon_{jil}\frac{\partial}{\partial x_i}\varepsilon_{jlk}\frac{\partial U_k}{\partial x_j}$$

$$\vec\nabla.\vec\nabla\vec U = (\partial_1\ \partial_2\ \partial_3)\begin{pmatrix}\partial_1 U_1 & & \\ \partial_2 U_1 & & \\ \partial_3 U_1 & & \end{pmatrix} = \begin{pmatrix}\partial_i\partial_i U_1\\ \partial_i\partial_i U_2\\ \partial_i\partial_i U_3\end{pmatrix}$$

$$\delta_{il}\delta_{lk}\frac{\partial}{\partial x_j}\left(\frac{\partial U_k}{\partial x_i}\right) + \varepsilon_{jil}\frac{\partial}{\partial x_i}(\vec\nabla\times\vec U)_l$$
$$\vec\nabla(\vec\nabla.\vec U) + \vec\nabla\times(\vec\nabla\times\vec U)$$

$$(\vec\nabla.\vec\nabla U)_j = \partial_i\partial_i U_j$$

$$\boxed{\nabla^2\vec U = \vec\nabla(\vec\nabla.\vec U) - \vec\nabla\times(\vec\nabla\times\vec U)}$$

$$\varepsilon_{jil}\,\varepsilon_{jmk} = \delta_{im}\delta_{lk} - \delta_{ik}\delta_{lm} \rightarrow l=m$$

$$\delta_{ik} = \delta_{il}\delta_{lk} - \varepsilon_{jil}\varepsilon_{jlk}$$

xix) $(\vec U.\vec\nabla)\vec U = \left(U_i\frac{\partial}{\partial x_i}\right)U_j = \left[\delta_{ik}\left(U_k\frac{\partial}{\partial x_i}\right)\right]U_j$

es un vector; escalar

$$= \delta_{il}\delta_{lk}\left(U_k\frac{\partial}{\partial x_i}\right)U_j - \varepsilon_{jil}\varepsilon_{jlk}U_k\frac{\partial U_j}{\partial x_i}$$

$$\left(U_i\frac{\partial}{\partial x_i}\right)U_j - \varepsilon_{jlk}\,\varepsilon_{lji}U_k\frac{\partial U_j}{\partial x_i}$$

$$\frac{\partial U_j}{\partial x_i}U_i - \varepsilon_{jlk}U_k(-\vec\nabla\times\vec U)_l$$

$$(\vec\nabla\vec U)\vec U + \varepsilon_{jkl}U_k(-\vec\nabla\times\vec U)_l$$

$$-\vec U\times(\vec\nabla\times\vec U)$$

usando xvii) es

$$\vec\nabla(\vec U.\vec U) = (\vec U\vec\nabla)\vec U + (\vec U\vec\nabla)\vec U + 2\,\vec U\times(\vec\nabla\times\vec U)$$

$$\frac{1}{2}\vec\nabla(\vec U\vec U) = (\vec U.\vec\nabla)\vec U + \vec U\times(\vec\nabla\times\vec U)$$

$$\boxed{(\vec U.\vec\nabla)\vec U = \frac{1}{2}\vec\nabla(\vec U.\vec U) - \vec U\times(\vec\nabla\times\vec U)}$$